Reg. No. .....

(2 pages)

K 6181

Name .....

COMBINED FIRST AND SECOND SEMESTER (S1 S2) B.Arch. DEGREE EXAMINATION, MAY/JUNE 2005

MATHEMATICS AND STATISTICS

(2003 scheme)

Time : Three Hours

Maximum : 100 Marks

Answer **all** *questions from Part A and* **one** *from each module of Part B.*

Part A

1. If $y = \dfrac{(ax + b)}{(cx + d)}$, show that $2 y_1 y_2 = 2 y_2^2$.

2. Find the co-ordinates of the centre of curvature of $(at^2, 2at)$ on the parabola $y^2 = 4ax$.

3. The mean and variance of 15 observations and 12 and 16 respectively. It was found that there were only 14 observations and the last observation "9" was used twice. Find the correct mean and variance.

4. Let X be a random variable taking values $\frac{1}{2}$ and $-\frac{1}{2}$ with probability $\frac{1}{2}$ and $\frac{1}{2}$ respectively. Obtain expectation of X and variance of X.

5. Write down the mathematical form of a general linear programming problem.

Part B

Module I

6. (a) Find the nth derivative of $\sin 2x \sin 3x$.

   (b) If $Y = (x^2 - 1)^n$, show that $(x^2 - 1) y_{n+2} + 2x y_{n+1} - n(n + 1) y_n = 0$.

   (c) Find the radius of curvature of $x^3 + y^3 = 3axy$ at $\left(\dfrac{3a}{2}, \dfrac{3a}{2}\right)$.

7. (a) Integrate $\displaystyle\int_0^1 x^4 \left(1 - x^2\right)^{3/2} dx$.

   (b) If $I_{m,n} = \displaystyle\int_0^{\pi/2} \cos^m x \cos nx \, dx$ show that $I_{m,n} = \dfrac{m}{m + n} I_{m-1, n-1}$.

   (c) Find the area included between the curve $y^2 (2a - x) = x^3$ and its asymptote.

Turn over

Module II

8. (a) Find the equation of the parabola whose focus is (1, 2) and directrix $x + 2y + 5 = 0$.

   (b) Find the centre foci and latus rectum of the ellipse $4x^2 + 9y^2 - 48x + 72y + 144 = 0$.

9. (a) Derive the standard equation to parabola.

   (b) Show that evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $(ax)^{2/3} + (by)^{2/3} = (a^2 - b^2)^{2/3}$.

Module III

10. Two samples of size 30 and 40 give the following :—

    Sample 1 Mean = 44.5 Variance = 12.8

    Sample 2 Mean = 48.5 Variance = 14.4

    Obtain the mean and variance of the combined sample.

11. Find the correlation coefficient of the following data :—

X	3.3	3.4	4.8	4.4	3.5	3.6	3.1	4.1
Y	13.1	14.2	15.8	16.3	14.9	13.8	12.8	16.3

Hence obtain regression of Y on X. If X = 4.5, what could be the possible value of Y ?

Module IV

12. (a) Write down the probability density function of the normal distribution. Briefly explain the properties of the normal distribution.

    (b) If X follows a binomial distribution with parameters $n = 10$ and $p = 0.03$, obtain

    (i) P (X = 7).

    (ii) P (X is less than 3).

    (iii) P (X takes at least a value 8).

13. (a) A random variable X follows Poisson distribution with parameter λ. Obtain the mean and variance.

    (b) X follows a normal distribution with mean 20 and variance 16. What is the probability that (i) $X > 25$; (ii) $X < 18$; (iii) $19 < X < 24$.

Module V

14. Solve the linear programming problem using graphical method of solution :

    Maximize $Z = 50x + 60y$

    subject to $2x + 3y \leq 1500$

    $3x + 2y \leq 1500$

    $x \leq 400$; $y \leq 400$

    $x \geq 0$; $y \geq 0$.

15. Solve the following linear programming problem using simplex method :—

    Maximize $45x_1 + 80x_2$

    subject to $5x_1 + 20x_2 \leq 400$

    $10x_1 + 15x_2 \leq 400$

    $x_1 \geq 0$; $x_2 \geq 0$.